Regulating entanglement production in multitrap Bose-Einstein condensates

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Abstract

A system of traps is considered, each containing a large number of Bose-condensed atoms. This ensemble of traps is subject to the action of an external modulating field generating nonequilibrium nonground-state condensates. When the frequency of the modulating field is in resonance with the transition frequency between two different topological coherent modes, each trap becomes an analog of a finite-level resonant atom. Similarly to the case of atoms in an electromagnetic resonant field, one can create entanglement between atomic traps subject to a common resonant modulating field generating higher coherent modes in each of the traps. A method is suggested for regulating entanglement production in such a system of multitrap and multimode Bose-Einstein condensates coupled through a common resonant modulating field. Several regimes of evolutional entanglement production, regulated by manipulating the external field, are illustrated by numerical calculations.

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I. INTRODUCTION

Entanglement is a notion that is important for quantum computation and quantum information processing [1–5]. Considering entanglement, one usually studies the systems of resonant atoms, possessing internal states, or spin assemblies [6–8]. As possible candidates for engineering entanglement, trapped ions have also been considered [1,2]. On the other hand, recent advances in atomic physics with neutral atoms, experiencing Bose-Einstein condensation [9–14], have led to several suggestions of employing these condensed atoms for producing entanglement. Thus, entanglement was shown to be feasible in Bose-condensed systems of atoms with two different internal states or with two collective modes [15,16]. Entanglement can be realized between the atoms from two clouds with different momenta, which arise in the process of coherent scattering of light from a Bose-Einstein condensate [17–20]. It is also possible to consider entanglement in Fermi-Bose mixtures [21–23] and in atom-molecule mixtures created by Feshbach resonance [24,25]. A review of the general properties of two or more entangled modes can be found in Ref. [26].

A promising and efficient way of generating several topological coherent modes in Bose-condensed systems is by means of the resonant modulation of the external trapping potential [27–30]. The properties of these coherent modes have also been studied in a number of works [31–45]. A Bose-condensed system with so generated coherent modes possesses a rich behaviour displaying several nontrivial effects, such as interference fringes [29,41], interference current [29,41], mode locking [27–29], dynamic transitions and critical phenomena [28,29,37], chaotic motion [30,44], harmonic generation and parametric conversion [30,44], and atomic squeezing [29,43]. It was mentioned [42,43] that a trap with several generated coherent modes is an analog of a finite-level resonant atom.

In the present paper, we consider not just a single trap, but an ensemble of many traps subject to the action of a common resonant modulating field, generating topological coherent modes in each of the traps. Such a system is equivalent to an ensemble of many resonant atoms subject to a common electromagnetic field. As is known [6], the atoms, resonantly interacting with a common field, become entangled, which is termed resonant entanglement. In the same way, there appears entanglement between the traps subject to a common resonant field generating coherent modes. This *resonant entanglement* is a particular case of entanglement between coherent modes [26]. One of possible ways of creating a multitrap Bose condensate could be by means of optical lattices, which should be sufficiently deep, so that a potential well around each lattice site would contain a large number of atoms, thus, representing a separate trap. Then the lattice as a whole should be shaken by an external modulating field, resonantly generating coherent modes in each of the lattice-site wells. In an experiment, the resonant field can be produced by a time-dependent modulation of magnetic fields or by laser beams [27–31]. The advantage of employing a deep optical lattice as a multitrap device is the possibility of varying the barriers between lattice sites and, hence, the feasibility of tuning the properties of the overall system.

A peculiar feature of entanglement, occurring in multitrap multimode Bose condensates, is that this is an entanglement between mesoscopic (or almost macroscopic) objects, as compared to the entanglement between atoms, which are microscopic objects. At the same time, a mesoscopic Bose condensate is a quantum object, described by a wave function satisfying a nonlinear Schrödinger equation. This unique combination of being mesoscopic

in size and quantum in its features makes Bose-Einstein condensate a specific matter. A multitrap collection of multimode condensates constitutes a novel system that can find applications in quantum computing and information processing.

Before going to a mathematical description, let us present in more detail the physical picture we keep in mind. A set of Bose-Einstein condensates can be created by loading ultracold atoms into a deep optical lattice. Thus a one-dimensional array of ^{87}Rb condensates was formed [46], with a period of 2.7×10^{-4} cm. The lattice was sufficiently deep, with the depth $V_0 \approx 2\pi \times 50$ kHz $\approx 3 \times 10^5$ Hz, or, in units of the recoil energy, $V_0/E_r \approx 600$. The total number of lattice sites was 30, each containing $\sim 10^4$ atoms. A similar experiment for ^{87}Rb condensates was reported in Ref. [47]. A deep one-dimensional optical lattice, with adjacent sites spaced by 5.3×10^{-4} cm, contained about 10^4 atoms. The number of lattice sites could be varied between 5 to 35, so that the number of condensed atoms in each site was between 100 to 10^3. In these experiments [46,47], the number of condensed atoms N in each of the lattice sites was very large, reaching $N \sim 10^4$. Each lattice site played the role of a trap containing a large number of completely Bose-condensed atoms in a well defined coherent state. But, because of the great depth of each site, the intersite tunneling was strongly suppressed. Such a state of matter is commonly called the Mott insulator. A specific feature of the insulating phase, formed in the experiments [46,47], is an enormous filling factor, with the number of atoms per lattice site being very large, $N \gg 1$. These insulating systems were probed [46,47] by releasing the atoms from their lattice, so that the wave packets of different condensates, emanating from separate lattice sites, could expand and overlap. A surprising discovery of these observations was that, contrary to what could be naively expected, there existed high-contrast interference fringes, which is commonly associated with the presence of coherence between different lattice sites. The interference-fringe contrast was indeed comparable to the results observed in the early interference experiments for two well-correlated condensates (see discussion in the review papers [10,14]). Similar results were also obtained for deep optical lattices with low filling factors, showing that some kind of coherence persists even deep in the insulating phase, preserving a good visibility of the interference patterns observed after free expansion [48]. The theoretical explanation of these interference effects shows that, though separate lattice sites may look to a first glance independent, the overall insulating phase is a highly correlated state of the many-body system, being a generically entangled state [49].

An additional degree of entanglement for such deep lattices, with large filling factors, is provided by subjecting the whole system to the action of an external resonant field, being *common* for all lattice sites. Since the resonant field is common for the total lattice, one has to treat the insulator plus the field as a whole, as a single complex object. This situation is completely analogous to what one has subjecting an ensemble of finite-level atoms to a common resonant field, realizing the so-called resonant entanglement [6]. There is a straightforward and direct analogy between these two cases [41,42,50].

A Bose-condensed gas can also be made equivalent to a finite-level system. For this purpose an external alternating field, acting on the Bose-Einstein condensate of trapped atoms, has to oscillate with a frequency ω close to the transition frequency ω_0 between the ground-state level and that of the chosen excited mode. Because of the nonlinearity of the stationary Gross-Pitaevskii equation, the spectrum of self-consistent collective modes is not equidistant. This spectrum, for different trap shapes, was calculated and analysed in Refs.

[10,27,34,38,39,42]. Applying several driving fields, with the appropriately tuned frequencies, a multimode system can be formed. Such a finite-level condensate is completely analogous to a finite-level atom subject to the action of a resonant electromagnetic field [41,42]. In the process of the resonant excitation of a Bose condensate, similar to the same procedure for an atom, there exist as well nonresonant transitions. However, their probability is much lower than that of the chosen resonant transition. The effect of nonresonant transitions was studied in Ref. [29], where it was shown that, for a typical setup, the occurrence of nonresonant transitions would destroy the condensate during the time approximately equal to the lifetime of atoms in a trap. Therefore, the resonant generation of nonground-state modes in a Bose condensate is a feasible procedure. These conclusions were obtained by employing accurate analytical calculations as well as by direct numerical simulations for the temporal Gross-Pitaevskii equation [30,44,45]. The resonant generation of a dipole mode was realized in experiment [32]. Vortices are the examples of the topological modes, which can be generated in the resonant way [31].

Each lattice site of a deep optical lattice represents a trap that may contain a large number of Bose-condensed atoms. According to experiments [46,47], this number can be as large as $N \sim 10^4$. A collection of the lattice sites composes a strongly correlated system. The correlation between different sites comes from three sources. First, the standard method of preparing the considered system is by initially condensing a large atomic cloud and then imposing on it a periodic lattice formed by laser beams. Hence, at the initial time all atoms compose a general condensed system in a single coherent state, after which they are loaded in different lattice sites [46–48]. Second, though the tunneling between deep lattice sites is small, but, nevertheless, it is finite, because of which phase coherence on short length scales persists even deep in the insulating phase [48]. Finally, when the overall lattice is subject to a *common* resonant field, the whole system of the lattice plus field must be treated as one complex object.

We show below that not merely one is able to generate entanglement in a multitrap Bose condensate, but, moreover, it is possible to effectively regulate the amount of the produced entanglement by manipulating with the applied resonant field. The potentialities of regulating the entanglement generation is what makes the considered method important for applications. Using numerical calculations, we present several regimes of regulated entanglement production, which illustrate wide possibilities of the suggested approach. In the last Section V, we discuss how the entanglement in the system can manifest itself in experiment.

Throughout the paper, the system of units is employed, where the Planck constant and Boltzmann constant are set to unity, $\hbar \equiv 1$, $k_B \equiv 1$.

II. SINGLE-TRAP COHERENT STATES

Before studying a collection of many traps, let us, first, give an accurate mathematical description of coherent states in a single trap. The intention of the present section is to introduce the definitions and notations characterizing a single trap, which will be used in the following section for a generalization to the case of many traps. Let a trap contain N Bose atoms at a very low temperature, when the whole system is Bose-condensed. The Bose-Einstein condensate, as is known [10,51], corresponds to a coherent state $|\eta >$ of a Fock space. The coherent state is an eigenstate of a destruction field operator, whose eigenvalue is

given by a coherent field $\eta(\mathbf{r})$. In the Fock space, a normalized coherent state is represented by a column

$$|\eta> = \left[\frac{e^{-N/2}}{\sqrt{n!}} \prod_{i=1}^{n} \eta(\mathbf{r}_i) \right] \tag{1}$$

with respect to the index $n = 0, 1, 2, \ldots$, enumerating the column rows. Equation (1) is the standard notation for a state in the Fock space [51]. A more detailed explanation of the notations and definitions, used in the paper, are given in Appendix. The scalar product of two coherent states is defined as

$$< \eta_1 | \eta_2 > \equiv \sum_{n=0}^{\infty} \frac{e^{-N}}{n!} (\eta_1, \eta_2)^n , \tag{2}$$

where the scalar product of two coherent fields is

$$(\eta_1, \eta_2) \equiv \int \eta_1^*(\mathbf{r}) \eta_2(\mathbf{r}) \, d\mathbf{r} . \tag{3}$$

The normalization conditions for the coherent states and fields are

$$< \eta | \eta > = 1 , \qquad (\eta, \eta) = N . \tag{4}$$

A coherent state is a ground state for a Bose-condensed system [10,51,52]. Here we keep in mind a dilute system of atoms confined by means of a trapping potential $U(\mathbf{r})$ and interacting through the local potential

$$\Phi(\mathbf{r}) = \Phi_0 \delta(\mathbf{r}) , \qquad \Phi_0 \equiv 4\pi \frac{a_s}{m} ,$$

in which a_s is a scattering length and m, atomic mass. The stationary coherent fields are defined by the Gross-Pitaevskii equation

$$\hat{H}[\eta_k] \, \eta_k(\mathbf{r}) = E_k \, \eta_k(\mathbf{r}) , \tag{5}$$

with the nonlinear Schrödinger Hamiltonian

$$\hat{H}[\eta] = - \frac{\nabla^2}{2m} + U(\mathbf{r}) + \Phi_0 |\eta|^2 . \tag{6}$$

The eigenproblem (5) yields a spectrum of the energies E_k, labelled by a multi-index k. The lowest energy $E_0 \equiv \min_k E_k$ corresponds to the usual Bose-Einstein condensate, with the related coherent field $\eta_0(\mathbf{r})$ being just the condensate wave function. The higher energies E_k correspond to the nonground-state condensates, with the wave functions $\eta_k(\mathbf{r})$ called the topological coherent modes [27–30]. Each mode is normalized as

$$(\eta_k, \eta_k) = N . \tag{7}$$

The modes are named topological, since they describe topologically different distributions of atoms $|\eta_k(\mathbf{r})|^2$. Recall a vortex that is the most known example of a topological mode.

For each coherent mode $\eta_k(\mathbf{r})$, one may construct, similarly to Eq. (1), a coherent state

$$|\eta_k> \equiv \left[\frac{e^{-N/2}}{\sqrt{n!}} \prod_{i=1}^{n} \eta_k(\mathbf{r}_i) \right] . \tag{8}$$

Equation (8), in the same way as Eq. (1), is the standard notation of a state in the Fock space. The right-hand side here denotes a column with an infinite number of rows enumerated by an index labelling the generic column element inside the square brackets [51]. These mode coherent states are normalized to one, $< \eta_k|\eta_k >= 1$, by analogy with Eq. (4). The coherent modes of Eq. (5), being defined by a nonlinear Schrödinger equation, are not necessarily orthogonal. Therefore the coherent states (8), with a scalar product

$$< \eta_k|\eta_p > = \exp\{-N + (\eta_k, \eta_p)\} , \tag{9}$$

look also nonorthogonal for $k \neq p$.

From the Cauchy-Schwartz inequality we know that

$$|(\eta_k, \eta_p)|^2 \leq (\eta_k, \eta_k)(\eta_p, \eta_p) ,$$

where the equality takes place if and only if $\eta_k(\mathbf{r}) = \eta_p(\mathbf{r})$. The labelling of the coherent modes is assumed to be done so that to avoid the degeneracy in the notation. That is, $\eta_k(\mathbf{r})$ can be equal to $\eta_p(\mathbf{r})$ then and only then, when k coincides with p. Hence, for different k and p, one has

$$|(\eta_k, \eta_p)| < N \qquad (k \neq p) .$$

Therefore, in the limit of a large number of particles, for the scalar product (9), we have

$$\lim_{N \to \infty} < \eta_k|\eta_p > = \delta_{kp} . \tag{10}$$

This is equivalent to saying that the states (8) are asymptotically orthogonal for $N \gg 1$. Recall that N here is the number of atoms in a single lattice site, which as is explained in the Introduction, can be made in experiments very large.

It is worth stressing that the coherent states (8), composed of the coherent fields $\eta_k(\mathbf{r})$, characterize collective states of an N-atomic system, and should not be confused with internal states of an atom. To avoid such a confusion, we deal here with atoms possessing no internal states.

III. MULTITRAP COHERENT STATES

Now let us consider an ensemble of L traps, each of which can possess M modes,

$$L = \sum_j 1 , \qquad M = \sum_k 1 . \tag{11}$$

The traps are enumerated with an index $j = 1, 2, \ldots, L$. As is discussed in the Introduction, such an ensemble of traps could be realized as an optical lattice, with the potential wells at each lattice site deep enough to incorporate a large number of atoms N. For sufficiently deep potential wells, the condensate wave functions are localized close to the related lattice sites [10,53], so that the coherent modes $\eta_{ik}(\mathbf{r})$ and $\eta_{jk}(\mathbf{r})$ of different traps practically do not overlap,

$$(\eta_{ik}, \eta_{jk}) = \delta_{ij}N \ . \tag{12}$$

This corresponds to an insulating lattice phase. For each coherent mode $\eta_{ik}(\mathbf{r})$, we construct a coherent state $|\eta_{ik}>$, by analogy with Eq. (8). These coherent states, because of Eqs. (10) and (12), are asymptotically orthogonal, such that

$$< \eta_{ik}|\eta_{jp} > \simeq \delta_{ij}\delta_{kp} \qquad (N \gg 1) \ . \tag{13}$$

For each set $\{|\eta_{jk}>\}$ of the coherent states $|\eta_{jk}>$ a resolution of unity

$$\sum_k |\eta_{jk} >< \eta_{jk}| \ \simeq \hat{1}$$

is asymptotically $(N \gg 1)$ valid, understood in the weak sense [10,52]. Thus, the set $\{|\eta_{jk}>\}$ forms a basis, possibly overcomplete.

Let us define a Hilbert space

$$\mathcal{H}_j \equiv \overline{\mathcal{L}}\{|\eta_{jk}>\} \tag{14}$$

as a closed liner envelope over the set $\{|\eta_{jk}>\}$. Then, $\mathcal{H}_j$ is a Hilbert space of the coherent states associated with a j-th trap. The total Hilbert space for an ensemble of L traps is the tensor product

$$\mathcal{H} = \bigotimes_j \mathcal{H}_j \ , \tag{15}$$

in which $j = 1, 2, \ldots, L$. The total space (15) contains entangled as well as disentangled states. We define the set of disentangled states

$$\mathcal{D} \equiv \{\bigotimes_j |\varphi_j > : \ |\varphi_j > \in \mathcal{H}_j\} \tag{16}$$

as a subset of $\mathcal{H}$, composed of all disentangled states, having the form of the tensor products $\bigotimes_j |\varphi_j >$.

When we deal with an absolutely equilibrium system, then, as is evident, all atoms condense to the lowest-energy state. This implies, assuming that all traps are identical, that in each of them solely the states $|\eta_{j0}>$ will be occupied, all other excited states being empty. In order to physically realize a nontrivial situation, it is necessary to invoke a mechanism for the creation of several coherent modes. Then, clearly, we have to consider a nonequilibrium system. An efficient mechanism for generating coherent modes has been suggested [27–30], based on a resonant modulation of the trapping potential. Dealing with a nonequilibrium system, the coherent states $|\eta(t)>$ from the Hilbert space (15) become functions of time t, generally, having the form

$$|\eta(t) > = \ \sum_k c_k(t) \bigotimes_j |\eta_{jk} > \ . \tag{17}$$

Here we assume that the same resonant pumping field acts on all traps, generating the coherent modes with the probability $|c_k(t)|^2$.

Varying the coefficients $c_k(t)$ in Eq. (17), different entangled states can be created. Maximal entanglement is achieved when $|c_k(t)|^2 = 1/M$. Thus, for $|c_k|^2 = 1/2$, $L = 2$ and $M = 2$, Eq. (17) would represent a Bell state. For $|c_k|^2 = 1/2$, $L > 2$ and $M = 2$, this would be a multicat state. And for $|c_k|^2 = 1/M$, $L > 2$, with $M > 2$, a multicat multimode state would arise.

The statistical operator, corresponding to the state (17), is

$$\hat{\rho}(t) = |\eta(t) >< \eta(t)| \; . \tag{18}$$

To quantify the entanglement generated in the system, we shall employ the measure of entanglement production introduced in Refs. [54,55]. The advantage of this measure is its generality, which allows for the usage of the measure for any multiparticle systems; for example, the four-particle entanglement [56,57] can be easily characterized. The general measure of entanglement production [54,55] can be defined for an arbitrary operator. Here we shall use it for the statistical operator (18).

Let us define a nonentangling counterpart of the operator (18) as

$$\hat{\rho}^{\otimes}(t) \equiv \bigotimes_j \hat{\rho}_j(t) \; , \tag{19}$$

where the partite operators are

$$\hat{\rho}_j(t) \equiv \mathrm{Tr}_{\{\mathcal{H}_{i \neq j}\}} \hat{\rho}(t) \; . \tag{20}$$

Entanglement, generated by the statistical operator (18), is quantified by the measure of entanglement production

$$\varepsilon(\hat{\rho}(t)) \equiv \log \; \frac{||\hat{\rho}(t)||_{\mathcal{D}}}{||\hat{\rho}^{\otimes}(t)||_{\mathcal{D}}} \; , \tag{21}$$

in which the logarithm is to the base 2 and

$$||\hat{\rho}(t)||_{\mathcal{D}} \equiv \sup_{f \in \mathcal{D}} ||\hat{\rho}(t) f|| \qquad (||f|| = 1)$$

is the operator spectral norm, associated with the corresponding vector norm, and restricted to the set of disentangled states (16).

For what follows, we shall need the notation

$$n_k(t) \equiv |c_k(t)|^2 \; , \tag{22}$$

characterizing the mode probabilities, or the fractional mode populations. As the partite operators (20), we get

$$\hat{\rho}_j(t) = \sum_k n_k(t)|\eta_{jk} >< \eta_{jk}| \; .$$

Also, we find the norms

$$||\hat{\rho}(t)||_{\mathcal{D}} = ||\hat{\rho}_j(t)||_{\mathcal{D}} = \sup_k n_k(t) \; , \qquad ||\hat{\rho}^{\otimes}(t)||_{\mathcal{D}} = [\sup_k n_k(t)]^L \; .$$

Then the measure (21) of entanglement, generated by the statistical operator (18), writes as

$$\varepsilon(\hat{\rho}(t)) = (1 - L) \log \sup_k n_k(t) \ . \tag{23}$$

This measure, for the present case, has a nice property of being expressed by the equation

$$\varepsilon(\hat{\rho}(t)) = (L - 1)\varepsilon_2(t) \tag{24}$$

through the bipartite measure

$$\varepsilon_2(t) \equiv -\log \sup_k n_k(t) \ . \tag{25}$$

The relation (24) here is valid for any number of traps L and any number of modes M. This relation is a particular concrete example of the fundamental property, formulated by Linden, Popescu, and Wootters [58,59], that "the parts determine the whole in a generic pure quantum state".

IV. REGULATING ENTANGLEMENT PRODUCTION

Entanglement is such a general effect that it exists practically for all condensed-matter systems, changing together with phase transitions [54,55]. However, in the majority of cases, it is not easy to govern the level of entanglement. To be practically useful, the system should allow for an efficient way of manipulating with the generated entanglement, as is the case of some simple spin systems [8].

Here we show how entanglement production can be regulated in a multitrap Bose-condensed system. To excite the higher coherent modes, it is necessary to use the method of resonant generation [27-30]. For this purpose, the system is subject to the action of an external modulating field

$$V(\mathbf{r}, t) = V_1(\mathbf{r}) \cos \omega t + V_2(\mathbf{r}) \sin \omega t \ , \tag{26}$$

whose frequency is tuned close to the transition frequency

$$\omega_0 \equiv E_1 - E_0 \tag{27}$$

between the ground-state mode, with an energy E_0, and an excited coherent mode, with an energy denoted by E_1. The resonance condition

$$\left| \frac{\Delta\omega}{\omega} \right| \ll 1 \qquad (\Delta\omega \equiv \omega - \omega_0) \tag{28}$$

is assumed to be valid. For a nonequilibrium system, one has to consider the time-dependent Gross-Pitaevskii equation

$$i \frac{\partial}{\partial t} \varphi(\mathbf{r}, t) = \left(\hat{H}[\varphi] + \hat{V} \right) \varphi(\mathbf{r}, t) \ , \tag{29}$$

in which the wave function is normalized to one, $(\varphi, \varphi) = 1$, the nonlinear Schrödinger Hamiltonian is

$$\hat{H}[\varphi] = -\frac{\nabla^2}{2m} + U(\mathbf{r}) + \Phi_0 N |\varphi|^2 , \qquad (30)$$

and $\hat{V} = V(\mathbf{r}, t)$ is the resonant field (26). We look for the solution of Eq. (29) in the form

$$\varphi(\mathbf{r}, t) = \sum_k c_k(t) \varphi_k(\mathbf{r}) e^{-iE_k t} , \qquad (31)$$

where the coefficients $c_k(t)$ change in time slower than the exponential $\exp(-iE_k t)$. The quantity $|c_k(t)|^2$ defines the probability of generating a k-th mode, or the fractional mode population. For these probabilities, the normalization condition holds

$$\sum_k |c_k(t)|^2 = 1 . \qquad (32)$$

Since we have assumed that all traps are identical, and the same resonant field acts on each of them, the probabilities $|c_k(t)|^2$ do not depend on the trap index.

A resonant field of type (26) generates one higher coherent mode. Wishing to create several topological coherent modes, one should apply several resonant fields with the frequencies tuned close to the transition frequencies of the chosen modes [30,44]. Here we consider a two-mode case, one of them being the ground-state mode and another, an excited coherent mode.

Substituting the form (31) into Eq. (29) and employing the averaging technique, we can derive the equations for the coefficients $c_k(t)$. This derivation, with all details and with a full mathematical foundation has been given earlier [27–30]. Below, we present only the results of this procedure. We shall need the notation for the interaction transition amplitude

$$\alpha_{kp} \equiv \Phi_0 N \left(|\varphi_k|^2, \ 2|\varphi_p|^2 - |\varphi_k|^2 \right) \qquad (33)$$

and the pumping-field transition amplitude

$$\beta_{kp} \equiv (\varphi_k, \hat{B} \varphi_p) , \qquad (34)$$

where $\hat{B} \equiv V_1(\mathbf{r}) - iV_2(\mathbf{r})$. For the two-mode case, it is convenient to introduce the average interaction amplitude

$$\alpha \equiv \frac{1}{2} \left(\alpha_{01} + \alpha_{10} \right) \qquad (35)$$

and to write the pumping-field amplitude in the form

$$\beta_{01} = \beta e^{i\gamma} , \qquad \beta \equiv |\beta_{01}| . \qquad (36)$$

We denote the effective detuning as

$$\delta \equiv \Delta + \frac{1}{2} \left(\alpha_{01} - \alpha_{10} \right) , \qquad (37)$$

where

$$\Delta \equiv \Delta\omega + \alpha_{00} - \alpha_{11} \ . \tag{38}$$

The fractional mode amplitude can be written as

$$c_k(t) = |c_k(t)| \exp\{i\pi_k(t)t\} \ , \tag{39}$$

where $\pi_k(t)$ is a real function represenating the mode phase. The final equations can be cast to the system of two real equations for the population difference

$$s(t) \equiv |c_1(t)|^2 - |c_0(t)|^2 \tag{40}$$

and for the effective phase difference

$$x(t) \equiv \pi_1(t) - \pi_0(t) + \gamma + \Delta \ . \tag{41}$$

These equations are

$$\frac{ds}{dt} = -\beta\sqrt{1 - s^2} \ \sin x \ ,$$

$$\frac{dx}{dt} = \alpha s + \frac{\beta s}{\sqrt{1 - s^2}} \ \cos x + \delta \ . \tag{42}$$

By scaling the time variable with α, one can see that the behaviour of the solutions to Eqs. (42) is defined by two dimensionless parameters

$$b \equiv \frac{\beta}{\alpha} \ , \qquad \varepsilon \equiv \frac{\delta}{\alpha} \ . \tag{43}$$

The former parameter characterizes the strength of the pumping field, and the second parameter describes an effective detuning from the resonance, which can always be made small, $|\varepsilon| \ll 1$. By solving Eqs. (42), one finds the fractional mode populations

$$n_0(t) = \frac{1 - s(t)}{2} \ , \qquad n_1(t) = \frac{1 + s(t)}{2} \ . \tag{44}$$

Finally, the entanglement generation is quantified by the measure (25) which, for the considered two-mode case, is

$$\varepsilon_2(t) = -\log_2 \sup\{n_0(t), n_1(t)\} \ . \tag{45}$$

This measure changes with time in the interval $0 \leq \varepsilon_2(t) \leq 1$, being maximal when $n_0 = n_1 = 1/2$.

Solving Eqs. (42), we suppose that at the initial time $t = 0$ solely the ground-state mode exists, so that $s(0) = -1$ and $x(0) = 0$. Hence, initially, $n_0(0) = 1$ and $n_1(0) = 0$, because of which $\varepsilon_2(0) = 0$. Then the pumping resonant field is switched on, which generates an excited coherent mode and produces entanglement. There are two ways of governing the amount of entanglement production.

First, one can vary the amplitude and frequency of the resonant pumping field, choosing by this the required parameters (43), whose values define two main regimes of an oscillatory behaviour of the population difference $s(t)$, and so, the regimes of $\varepsilon_2(t)$. These are the mode-locked and mode-unlocked regimes [27–30,34,37,41–44]. We solve numerically Eqs. (42) and calculate the measure of entanglement production (45). Keeping in mind that the detuning can always be made small, we set $\delta = 0$. Then the value $b_c = 0.497764$ is the critical point for the change of the dynamical regimes. Below b_c, the measure (45) oscillates with time, never reaching one. When the parameter b, defined in Eq. (43), equals the critical point $b = b_c$, then the oscillating $\varepsilon_2(t)$ reaches one. For the dimensionless amplitude of the pumping field $b > b_c$, the oscillations of $\varepsilon_2(t)$ are always in the interval between 0 and 1. But the oscillation period sensitively depends on the value of b. Thus, for $b = 0.5$, just a little above b_c, the period of oscillations is more than twice shorter than that for $b = b_c$. The period for $b = 0.7$ is about eight times shorter than for $b = b_c$. Thus, by varying the amplitude of the pumping resonant field, we can strongly influence the evolution of $\varepsilon_2(t)$ both in its amplitude and period of oscillations.

There is one more very interesting way of regulating entanglement generation, which can be done by switching on and off the applied resonant field. Recall that this alternating field can be easily produced by modulating the magnetic field forming the trapping potential in magnetic traps or by varying the laser intensity in optical traps. Then, it is possible to create various sequences of pulses for $\varepsilon_2(t)$. For example, by switching on and off the resonant field in a periodic manner, we may form equidistant pulses of $\varepsilon_2(t)$, with all pulses having the same shape, as is demonstrated in Fig. 1. But we can also switch on and off the pumping field at different time intervals, thus, forming nonequidistant pulses, as is shown in Fig. 2. The possibility of creating very different pulses is illustrated in Fig. 3. Regulating entanglement production by means of a manipulation with the resonant pumping field, it is feasible to organize a kind of the Morse alphabet.

V. RELATION TO TIME-OF-FLIGHT EXPERIMENTS

It is important to add an idea how the entanglement generated in the system could manifest itself in order to verify it in a measurement. This can be done by observing the expansion of atomic clouds released from the optical lattice. The absorption images and interference fringes, observed in such time-of-flight experiments, are known to strongly depend on the initial conditions of the atoms before their release. If the initial spatial distribution of atoms in one case is essentially different from that in another case, the interference pictures for such two cases will be noticeably different. The generation of topological coherent modes, from one side, is directly related to the level of the induced entanglement and, from another side, drastically changes the spatial distribution of atoms. Recall that the considered modes are termed topological exactly because of their principally distinct from each other spatial shapes [27–30]. Radically differing initial conditions, due to differently prepared modes, will inevitably result in distinguishable one from another absorption images. As is seen from Eq. (17), if there exists just one mode, say the ground-state mode, so that $n_0 = |c_k|^2 = 1$, hence the fractional populations for all other modes are zero, then $|\eta(t)>$ in Eq. (17) is a product state, with no entanglement. Respectively, the measures (24), (25), and (45) are identically zero. By generating excited coherent modes,

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one makes the state (17) entangled. For instance, in the case of two modes, the maximal entanglement is achieved for $|c_k| = 1/\sqrt{2}$, when Eq. (17) represents a multicat state. Then the measure of entanglement production (24) reaches its maximal value $(L-1)\log 2 = L-1$, assuming that the logarithm is to the base two. Then the measures (25) and (45) are also maximal, being equal to one.

Suppose that at the time t_0 atoms were released from their traps. In the time-of-flight experiments one measures the atomic distribution

$$\tilde{\rho}(\mathbf{p}) \equiv \int \rho(\mathbf{r}, \mathbf{r}', t_0)\, e^{-i\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')}\, d\mathbf{r}\, d\mathbf{r}' , \qquad (46)$$

in which $\mathbf{p} = m\mathbf{r}/(t - t_0)$ and $\rho(\mathbf{r}, \mathbf{r}', t_0)$ is the first-order density matrix of atoms just before their expansion. The distribution (46) is normalized to the total number of particles NL, with N being the number of atoms in each lattice site and L, the number of sites, so that

$$\int \tilde{\rho}(\mathbf{p})\, \frac{d\mathbf{p}}{(2\pi)^3} = \int \rho(\mathbf{r}, t_0)\, d\mathbf{r} = NL , \qquad (47)$$

where $\rho(\mathbf{r}, t_0) = \rho(\mathbf{r}, \mathbf{r}, t_0)$ is the density of atoms before their release.

The field operator, acting on the total Hilbert space (15), can be represented as a direct sum

$$\psi(\mathbf{r}) = \oplus_j \psi_j(\mathbf{r}) \qquad (48)$$

of the operators $\psi_j(\mathbf{r})$ acting on the spaces $\mathcal{H}_j$. By the definition of the coherent states, one has

$$\psi_j(\mathbf{r})|\eta_{jk}> = \eta_{jk}(\mathbf{r})|\eta_{jk}> . \qquad (49)$$

For the atomic system in the state (17), the density matrix at the moment t_0 is

$$\rho(\mathbf{r}, \mathbf{r}', t_0) = <\eta(t_0)|\psi^\dagger(\mathbf{r}')\psi(\mathbf{r})|\eta(t_0)> . \qquad (50)$$

From here, using Eqs. (48) and (49), we find

$$\rho(\mathbf{r}, \mathbf{r}', t_0) = \sum_k \sum_{ij} |c_k(t_0)|^2 \eta_{ik}(\mathbf{r})\, \eta_{jk}^*(\mathbf{r}') . \qquad (51)$$

Substituting this into Eq. (46), we get

$$\tilde{\rho}(\mathbf{p}) = \sum_k \sum_{ij} n_k(t_0)\, \tilde{\eta}_{ik}^*(\mathbf{p})\, \tilde{\eta}_{jk}(\mathbf{p}) , \qquad (52)$$

were the Fourier transform

$$\tilde{\eta}_{jk}(\mathbf{p}) \equiv \int \eta_{jk}(\mathbf{r})\, e^{-i\mathbf{p} \cdot \mathbf{r}}\, d\mathbf{r} \qquad (53)$$

is introduced. Defining also the partial distribution

$$\tilde{\rho}_k(\mathbf{p}) \equiv \sum_{ij} \tilde{\eta}_{ik}^*(\mathbf{p})\, \tilde{\eta}_{jk}(\mathbf{p}) , \qquad (54)$$

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we finally obtain

$$\tilde{\rho}(\mathbf{p}) = \sum_k n_k(t_0) \, \tilde{\rho}_k(\mathbf{p}) \,. \tag{55}$$

When there is a sole mode, say when $n_0(t_0) = 1$, then there is no entanglement, and the measured distribution of atoms in the time-of-flight experiment is $\tilde{\rho}_0(\mathbf{p})$. The generation of excited modes creates entanglement and at the same time essentially changes the distribution of expanding atoms (55).

To give a more explicit illustration, let us consider a one-dimensional array of lattice sites in the tight-binding approximation. The ground-state wave function of atoms in a j-th site can be described by the localized orbital

$$\eta_{j0}(z) = \frac{1}{\pi^{1/4}} \, \sqrt{\frac{N}{l_0}} \, \exp\left\{-\frac{1}{2}\left(\frac{z - a_j}{l_0}\right)^2\right\} \,, \tag{56}$$

in which $a_j \equiv ja$, with a being a lattice spacing and $j = 0, \pm 1, \pm 2, \ldots$. The effective width of the atomic cloud, given by l_0, is generally defined through a variational procedure taking into account atomic interactions [10,27,42]. The Fourier transform of Eq. (56) is

$$\tilde{\eta}_{j0}(p) = (4\pi)^{1/4}\sqrt{Nl_0} \, \exp\left(-\frac{1}{2} \, p^2 l_0^2 - ipa_j\right) \,. \tag{57}$$

For the corresponding distribution (54), we get

$$\tilde{\rho}_0(p) = 2\sqrt{\pi} \, NLl_0 \exp\left(-p^2 l_0^2\right)\left(1 + \frac{2}{L}\sum_{i<j} \cos pa_{ij}\right) \,, \tag{58}$$

where $a_{ij} \equiv a_i - a_j$. This is what one would observe in the time-of-flight experiment, when all atoms before the release are in the ground state and there is no entanglement.

Suppose now that, in addition to the ground-state mode, the first excited mode is generated having in a j-th site the wave function

$$\eta_{j1}(z) = \frac{1}{\pi^{1/4}} \, \sqrt{\frac{2N}{l_1}} \left(\frac{z - a_j}{l_1}\right) \exp\left\{-\frac{1}{2}\left(\frac{z - a_j}{l_1}\right)^2\right\} \,. \tag{59}$$

Here l_1 is an effective width, which can be defined through a variational procedure [10,27,42]. Calculating the Fourieir transform of Eq. (59), we obtain

$$\tilde{\eta}_{j1}(p) = -i2\pi^{1/4}\sqrt{Nl_1} \, pl_1 \exp\left(-\frac{1}{2} \, p^2 l_1^2 - ipa_j\right) \,. \tag{60}$$

Consequently, the related atomic distribution is

$$\tilde{\rho}_1(p) = 4\sqrt{\pi} \, NLp^2 l_1^2 \exp\left(-p^2 l_1^2\right)\left(1 + \frac{2}{L}\sum_{i<j} \cos pa_{ij}\right) \,. \tag{61}$$

The total observable atomic distribution in the two-mode case is the sum

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$$\tilde{\rho}(p) = n_0(t_0)\tilde{\rho}_0(p) + n_1(t_0)\tilde{\rho}_1(p) . \tag{62}$$

If both $n_0(t_0)$ as well as $n_1(t_0)$ are nonzero, then entanglement is generated in the system. At the same time, the measurable distribution (62) is essentially different from the situation when just one term $\tilde{\rho}_0(p)$ is present. In the particular case of two sites, when $L = 2$, the terms in Eq. (62) are defined by

$$\tilde{\rho}_0(p) = 4\sqrt{\pi}\, Nl_0 \exp\left(-p^2 l_0^2\right)(1 + \cos pa) , \tag{63}$$

where a is the intersite distance, and

$$\tilde{\rho}_1(p) = 8\sqrt{\pi}\, Np^2 l_1^3 \exp\left(-p^2 l_1^2\right)(1 + \cos pa) . \tag{64}$$

It is evident that the sum (62) of the distributions (63) and (64) is drastically different from the sole term (63). This difference is especially noticeable at the point $p = 0$, where

$$\tilde{\rho}(0) = n_0(t_0)\tilde{\rho}_0(0) = 8\sqrt{\pi}\, Nl_0 n_0(t_0) . \tag{65}$$

Value (65) diminishes as soon as the second mode is generated, so that $n_0(t_0)$ becomes less than one.

In conclusion, a multitrap ensemble of multimode Bose-Einstein condensates, subject to the action of a *common* resonant field, is analogous to a system of finite-level atoms in a common resonant electromagnetic field. A multitrap system can be formed, e.g., as an optical lattice with deep potential wells, incorporating many atoms around each lattice site. In the multitrap multimode condensate a high level of entanglement can be achieved. By varying the amplitude and frequency of the pumping resonant field, different regimes of evolutional entanglement can be realized. Moreover, by switching on and off the pumping field in various ways, it is feasible to create entanglement pulses of arbitrary length and composing arbitrary sequences of *punctuated entanglement generation*. Such a high level of admissible manipulation with and regulating of entanglement can, probably, be useful for information processing and quantum computing.

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Appendix. Representation of coherent states in the Fock space.

In this Appendix, we give, according to the referee's advice, an explanation of the main notations and definitions, used in the paper, which are related to the representation of coherent states in the Fock space. We mention here only the basic facts. Substantially more details, with an accurate mathematical foundation, cane be found in the very good book by Berezin [51]. Of course, there exist many other publications expounding this material, for instance, the classical textbook by Schweber [60]. Our notations are close to those employed in these books [51,60].

Recall, first, that the Fock space is a direct sum

$$\mathcal{F} = \oplus_{n=0}^{\infty} \mathcal{H}_n$$

of n-particle Hilbert spaces $\mathcal{H}_n$ composed of symmetric or antisymmetric functions, according to whether the Bose-Einstein or Fermi-Dirac statistics are considered. For spinless bosons, treated in the paper, the wave function $f_n(\mathbf{r}_1, \ldots, \mathbf{r}_n)$ is symmetric with respect to the permutations of the spatial variables $\mathbf{r}_i$, for all $i = 1, 2, \ldots, n$. The n-particle function $f_n(\mathbf{r}_1, \ldots, \mathbf{r}_n)$ pertains to the Hilbert space $\mathcal{H}_n$. A state φ of the Fock space $\mathcal{F}$ is represented by the column

$$\varphi = \begin{bmatrix} f_0 \\ f_1(\mathbf{r}_1) \\ f_2(\mathbf{r}_1, \mathbf{r}_2) \\ f_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \\ \vdots \\ f_n(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n) \\ \vdots \end{bmatrix}$$

It is clear that to write down each time such a column would be a too cumbersome way. Therefore one commonly shortens the notation, representing the state $\varphi \in \mathcal{F}$ as

$$\varphi = [f_n(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)] \, ,$$

where only the generic element of the above column is written, with the index $n = 0, 1, 2, \ldots$ enumerating the rows of the column. Thus, the index n at the n-th row of the state φ tells that the n-particle wave function $f_n(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)$ is implied. The scalar product of two states φ, φ' of the Fock space $\mathcal{F}$ is defined as

$$(\varphi, \varphi') \equiv \sum_{n=0}^{\infty} (f_n, f'_n) \, ,$$

where

$$(f_n, f'_n) \equiv \int f_n^*(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n) f'_n(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n) \, d\mathbf{r}_1 \ldots d\mathbf{r}_n$$

is a scalar product in the n-particle Hilbert space $\mathcal{H}_n$. When a particular kind of wave functions $f_n(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)$ is kept in mind, this is often emphasized by denoting the corresponding state in the Fock space as

$$\varphi \equiv |f> = [f_n(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)] \, .$$

Coherent states $|\eta>$ in the Fock space are defined as the eigenvectors of the annihilation field operator $\psi(\mathbf{r})$, so that

$$\psi(\mathbf{r})|\eta> = \eta(\mathbf{r})|\eta> ,$$

where the coherent field $\eta(\mathbf{r})$ plays the role of the eigenvalue. Being a member of the Fock space, a coherent state is also represented by a column

$$|\eta> = \begin{bmatrix} \eta_0 \\ \eta_1(\mathbf{r}_1) \\ \eta_2(\mathbf{r}_1, \mathbf{r}_2) \\ \vdots \\ \eta_n(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n) \\ \vdots \end{bmatrix} ,$$

which is equivalently denoted by the accepted short-hand notation as

$$|\eta> = [\eta_n(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n)] .$$

From the definition of the coherent states as the eigenvectors of the field operator $\psi(\mathbf{r})$, and using the action of the latter yielding

$$\psi(\mathbf{r})|\eta> = \left[\sqrt{n+1}\, \eta_n(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n, \mathbf{r}) \right] ,$$

one gets the recursion relation

$$\sqrt{n+1}\, \eta_{n+1}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n, \mathbf{r}) = \eta(\mathbf{r})\, \eta_n(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n) .$$

Solving this equation by repeated iterations, one finds

$$\eta_n(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n) = \frac{\eta_0}{\sqrt{n!}} \prod_{i=1}^n \eta(\mathbf{r}_i) .$$

The normalization condition $<\eta|\eta> = 1$ gives

$$|\eta_0| = \exp\left\{ -\frac{1}{2}(\eta, \eta) \right\} .$$

Assuming that the coherent field $\eta(\mathbf{r})$ is normalized to the number of particles N, one has

$$(\eta, \eta) \equiv \int \eta^*(\mathbf{r})\eta(\mathbf{r})\, d\mathbf{r} = N .$$

In this way, for a coherent state $|\eta>$, up to a global phase factor, one obtains

$$|\eta> = e^{-N/2} \begin{bmatrix} 1 \\ \eta(\mathbf{r}_1) \\ \frac{1}{\sqrt{2!}}\, \eta(\mathbf{r}_1)\eta(\mathbf{r}_2) \\ \frac{1}{\sqrt{3!}}\, \eta(\mathbf{r}_1)\eta(\mathbf{r}_2)\eta(\mathbf{r}_3) \\ \vdots \\ \frac{1}{\sqrt{n!}}\, \eta(\mathbf{r}_1)\eta(\mathbf{r}_2)\dots\eta(\mathbf{r}_n) \\ \vdots \end{bmatrix} .$$

This is exactly the coherent state (1) written down in the accepted short-hand notation.

As is seen, to define the whole coherent state $|\eta>$, one needs to find the coherent field $\eta(\mathbf{r})$. The latter, for the considered case of a dilute Bose gas with contact interactions, is a solution of the Gross-Pitaevskii equation.
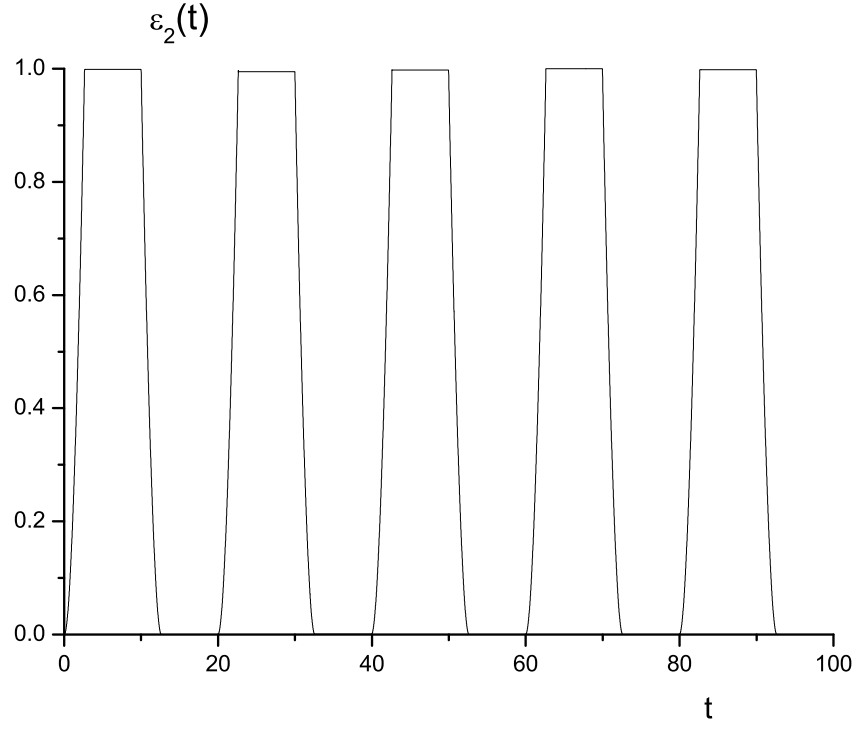
Figure Captions

Fig. 1. Regulated equidistant pulses of $\varepsilon_2(t)$, formed by switching on and off the resonant field, with $b = 0.7$, so that $\varepsilon_2(t)$ equals one during the time intervals $\Delta t = 7.35$ (in units of α^{-1}), and it equals zero during the same intervals $\Delta t = 7.35$. Time is measured in units of α^{-1}.

Fig. 2. Nonequidistant pulses of $\varepsilon_2(t)$, created by switching on and off the pumping field, with $b = 0.7$, at nonequal time intervals. Time is measured in units of α^{-1}.

Fig. 3. Regulated pulses of $\varepsilon_2(t)$, for the same $b = 0.7$, as in Fig. 2, but for essentially different moments of switching on and off the pumping field. Time is measured in units of α^{-1}.

FIG. 1. Regulated equidistant pulses of $\varepsilon_2(t)$, formed by switching on and off the resonant field, with $b = 0.7$, so that $\varepsilon_2(t)$ equals one during the time intervals $\Delta t = 7.35$ (in units of α^{-1}), and it equals zero during the same intervals $\Delta t = 7.35$. Time is measured in units of α^{-1}.
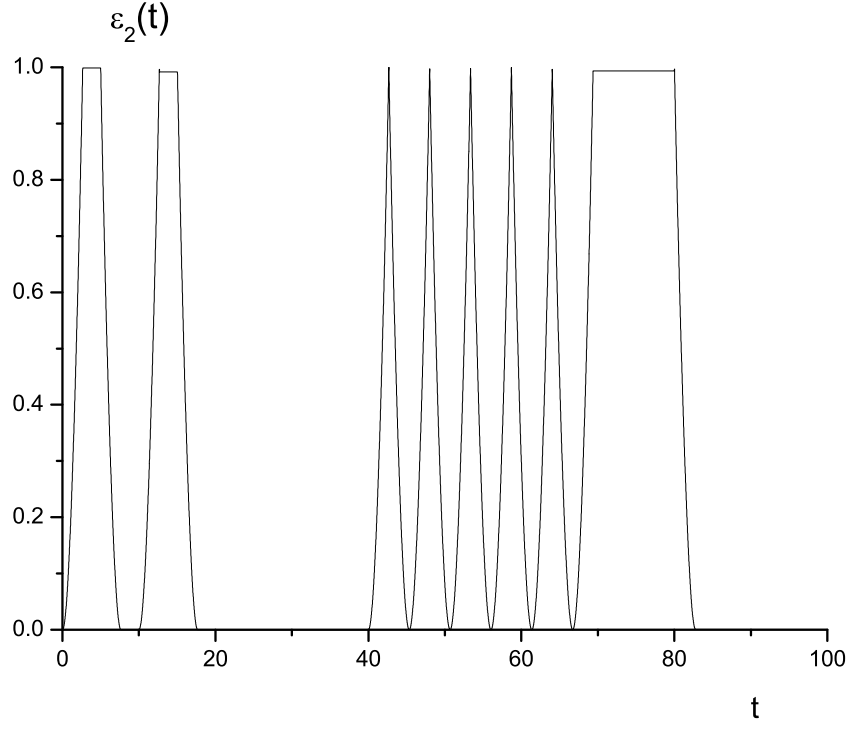
FIG. 2. Nonequidistant pulses of $\varepsilon_2(t)$, created by switching on and off the pumping field, with $b = 0.7$, at nonequal time intervals. Time is measured in units of α^{-1}.
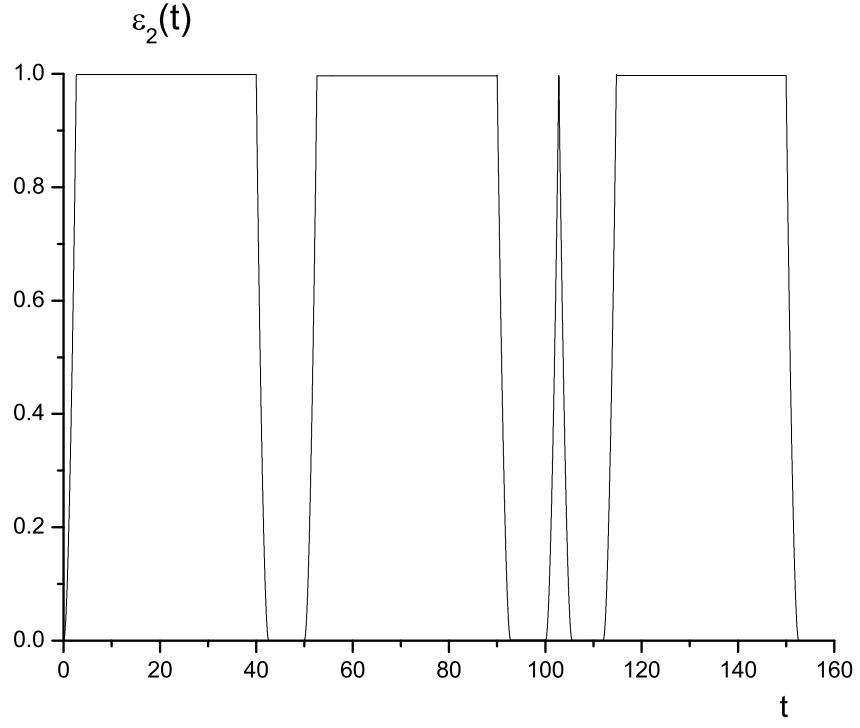
FIG. 3. Regulated pulses of $\varepsilon_2(t)$, for the same $b = 0.7$, as in Fig. 2, but for essentially different moments of switching on and off the pumping field. Time is measured in units of α^{-1}.